

# Sistemas difusos evolutivos en el modelamiento de fenómenos complejos

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# Agenda

- Sistemas difusos
- Modelado difuso evolutivo
- Casos de aplicación
- Discusión

“Not to be absolutely certain is, I think, one of the essential things in rationality” *Bertrand Russell*

## **SISTEMAS DIFUSOS**

# Lenguaje e incertidumbre

**SI el apartamento está cerca y el arriendo es barato  
ENTONCES**

**Podré ahorrar mucho dinero.**

**SI el apartamento está cerca y el arriendo es caro  
ENTONCES**

**Tan solo podré ahorrar algo de dinero**

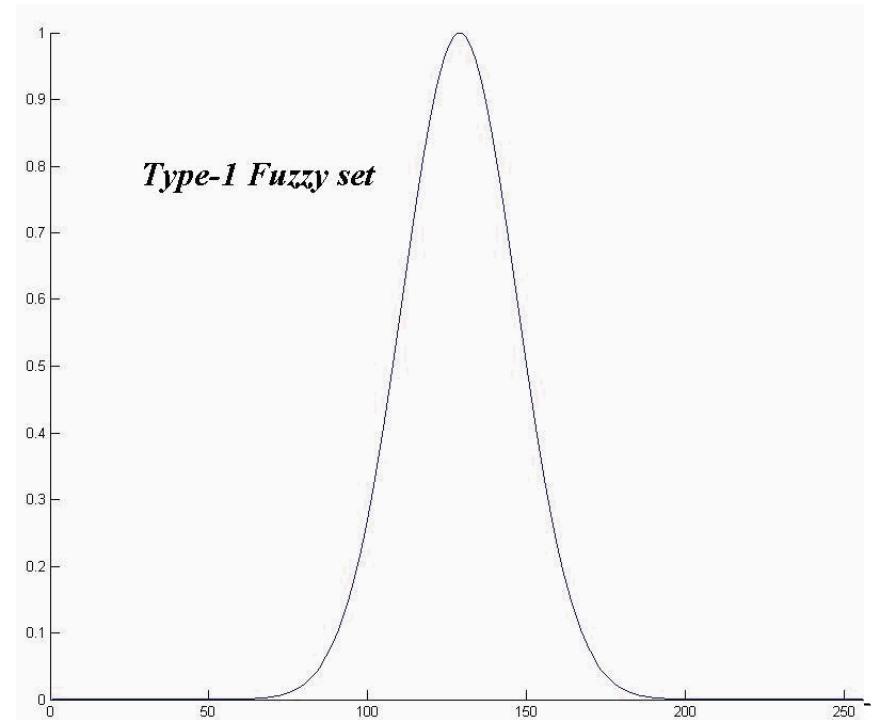
# Incertidumbre y conjuntos difusos



Experto



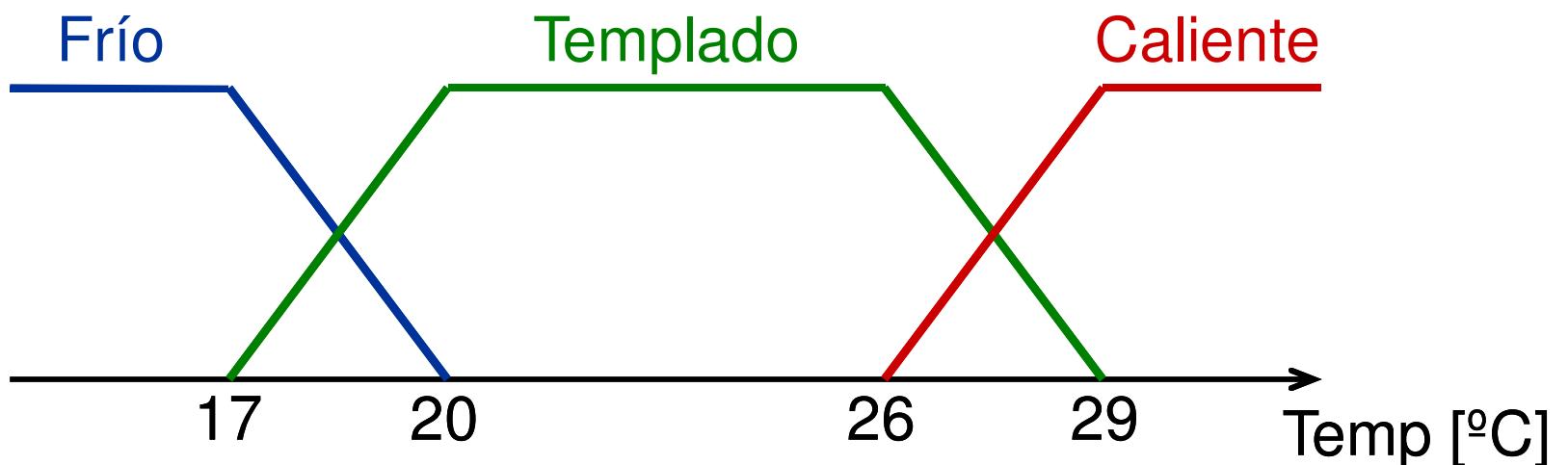
su definición  
de  
*adecuada*  
(Etiqueta lingüística)



Temperatura ( C )  
Variable Lingüistica

Generalmente las palabras (Etiquetas lingüísticas) tienen significados difusos

# Modelando la incertidumbre: variables lingüísticas y conjuntos difusos



- L.A. Zadeh, “**Fuzzy sets**”, Inf. Control 8, 338-353, **1965**

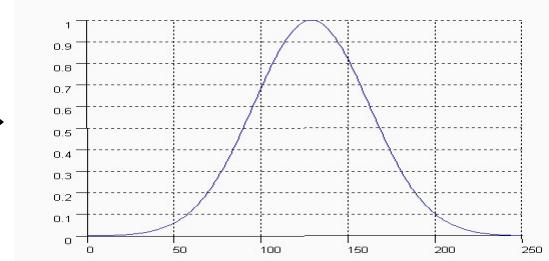
Carlos A. Peña, Evolutionary Fuzzy Modelling, 2000

# Incertidumbre y conjuntos difusos

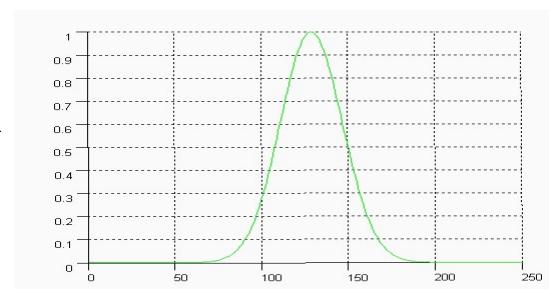
- **Vaguedad** = imprecisión lingüística  
e.g. El café esta medio tibio
- **Diversidad** = imprecisión en la información  
e.g. Es probable que mañana llueva.
- **Desacuerdo** = conflicto o discrepancia.  
e.g. Por favor llegue temprano!

# Incertidumbre y conjuntos difusos

Experto (1)  
adecuada



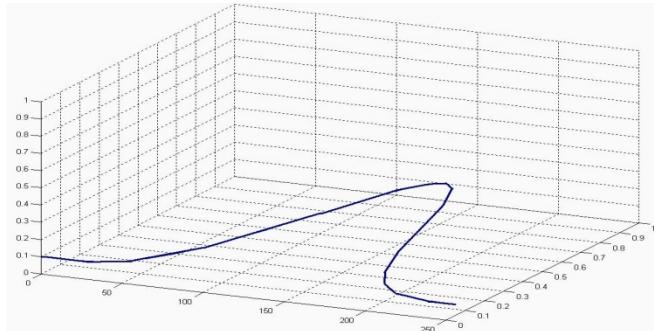
Experto (2)  
adecuada



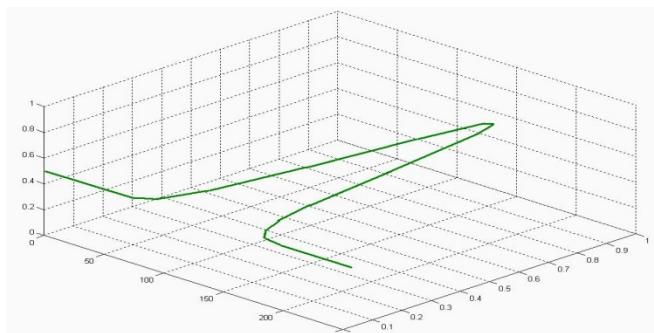
Experto (M)  
adecuada



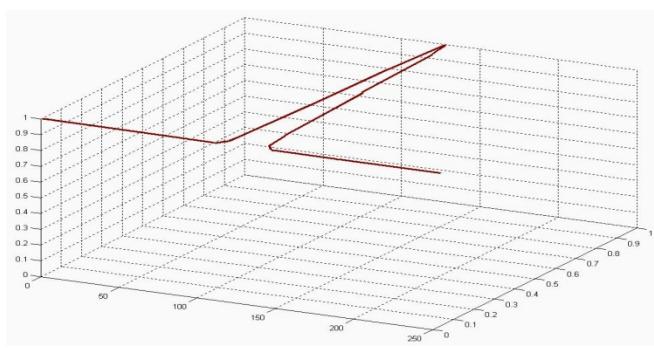
# Representación de la incertidumbre



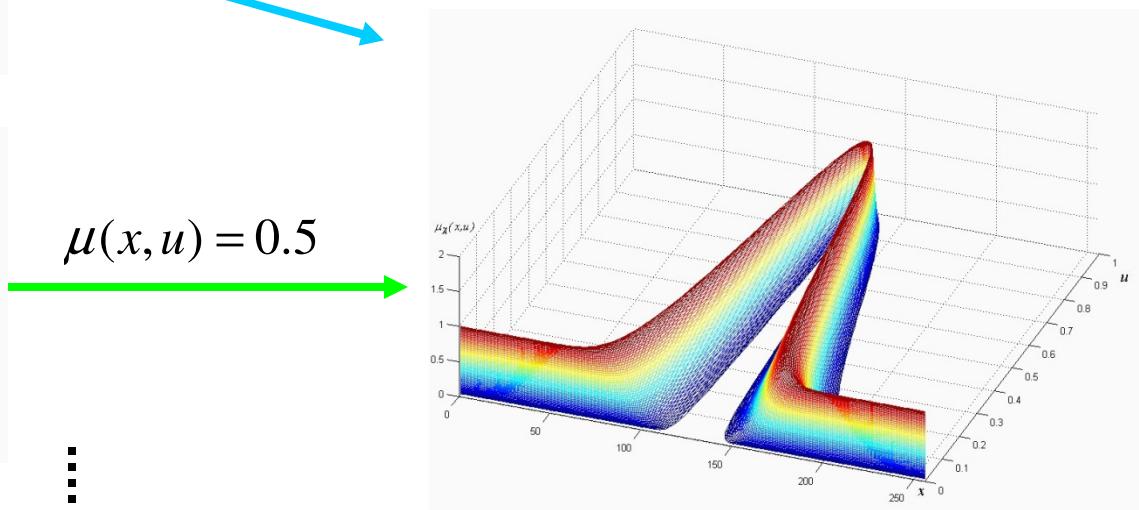
$$\mu(x,u) = 0.1$$



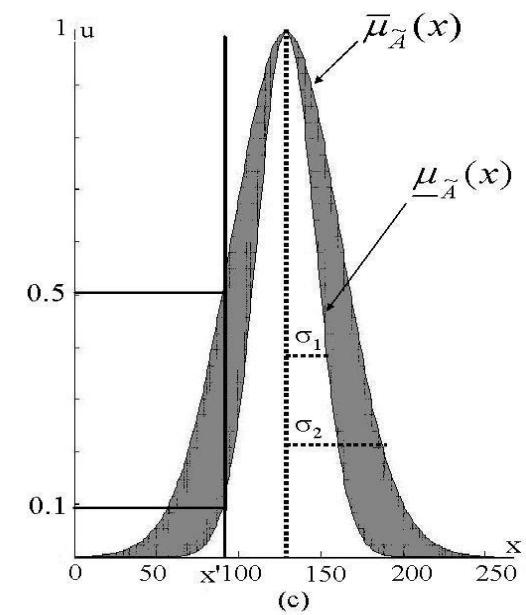
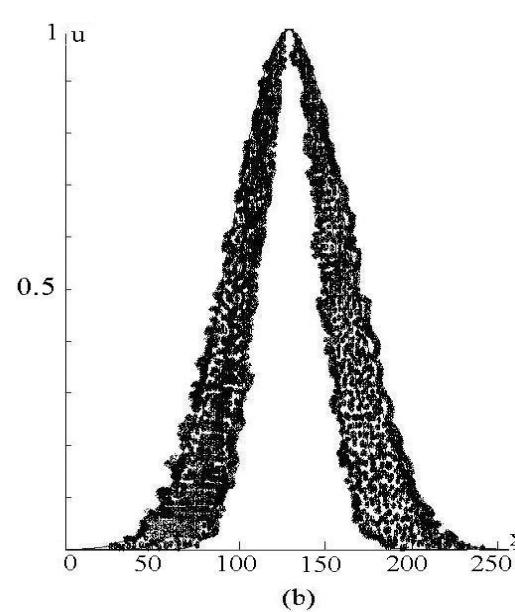
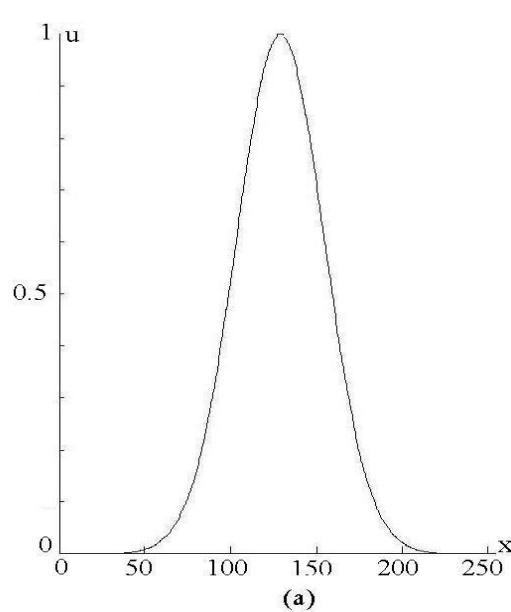
$$\mu(x,u) = 0.5$$



$$\mu(x,u) = 1$$



# Huella de incertidumbre



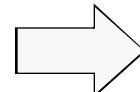
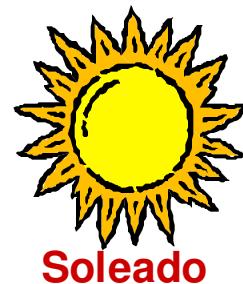
- a. Función de pertenencia Gaussiana
- b. Incertidumbre alrededor de la función de pertenencia
- c. Huella de incertidumbre

# Modelando la incertidumbre: reglas difusas

El problema de predicción de turistas.



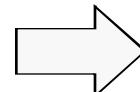
OR



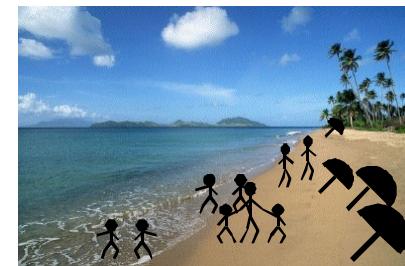
Alto



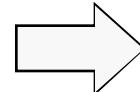
AND



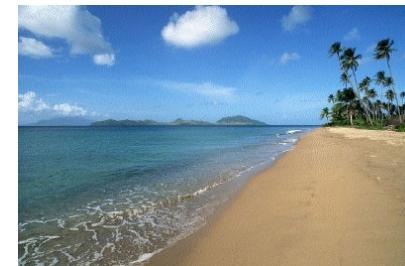
Medio



OR

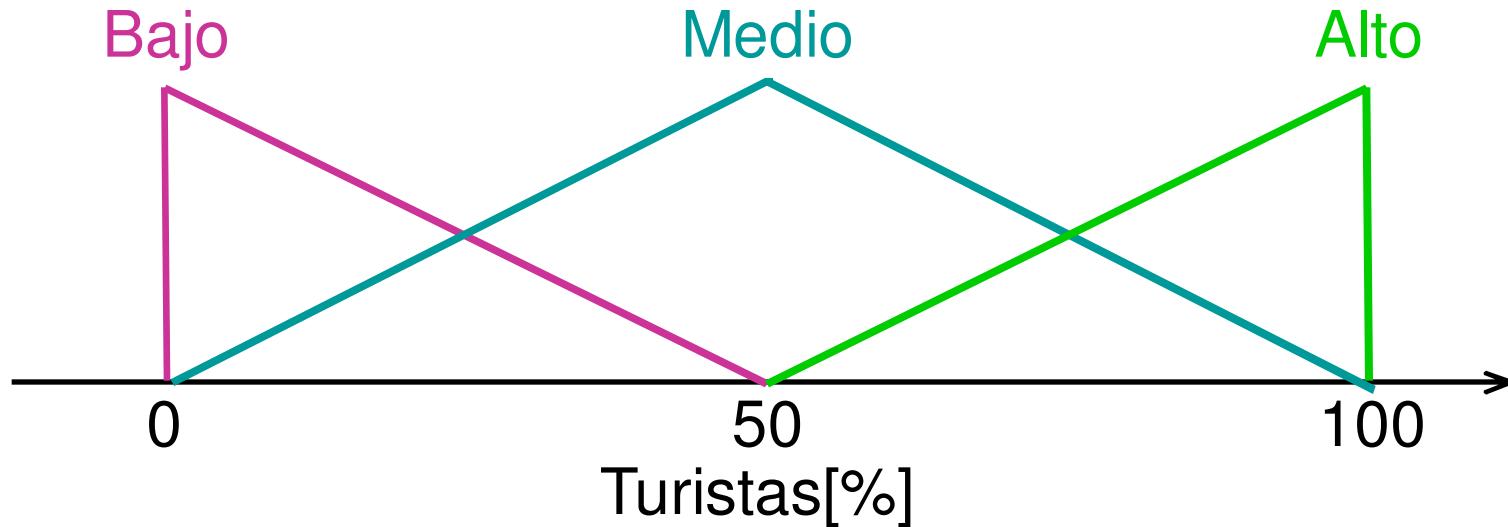
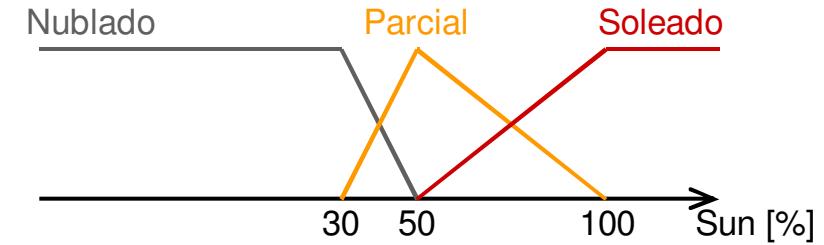
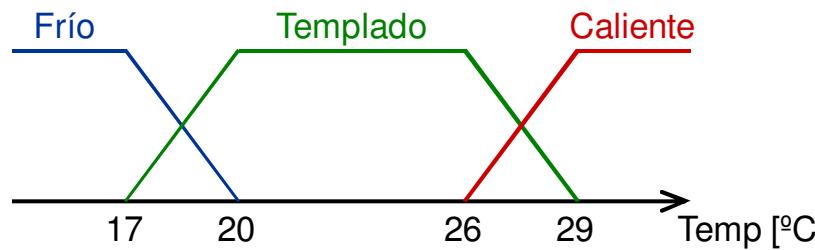


bajo



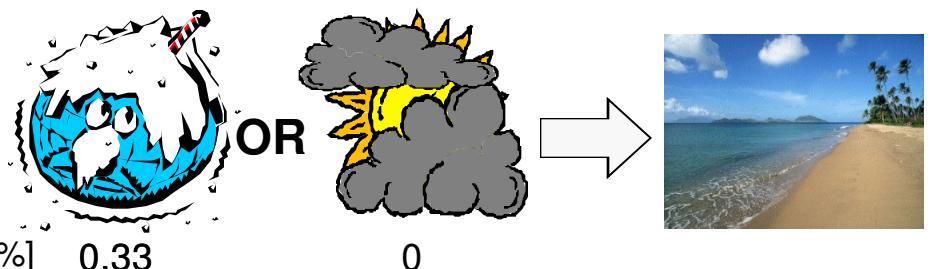
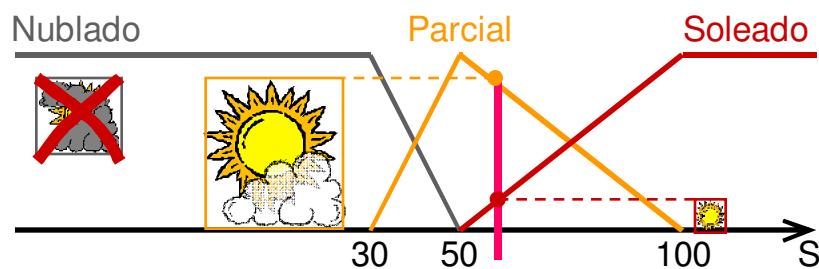
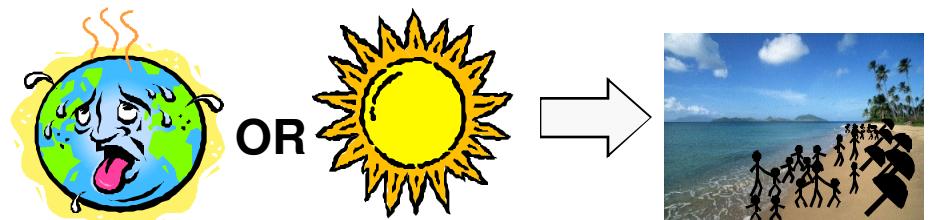
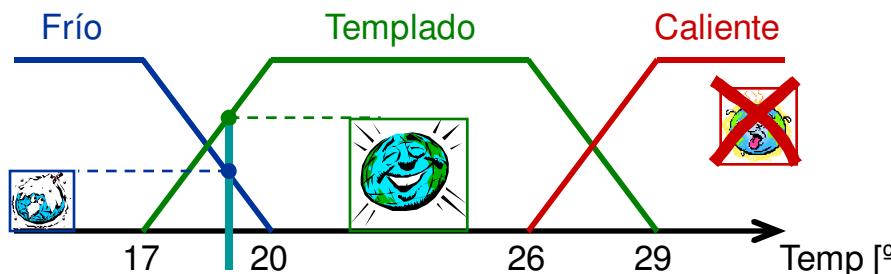
Sunshine

# Variables lingüísticas



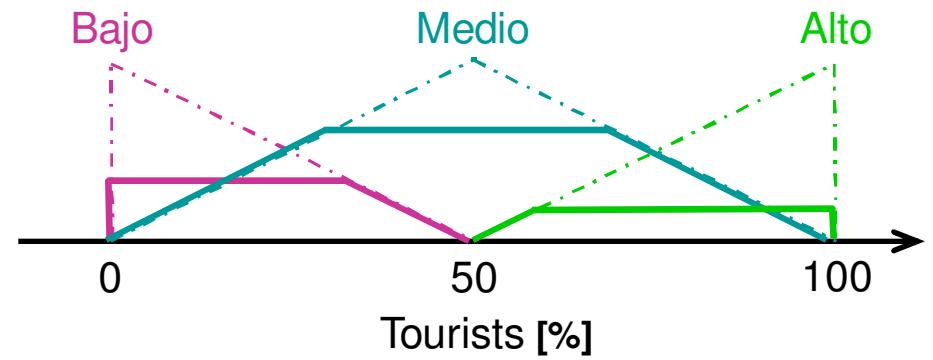
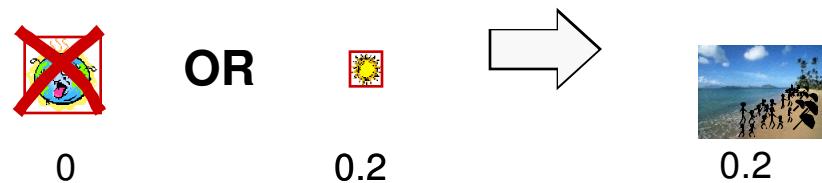
# Inferencia difusa

Pronóstico del clima ( temperatura:19º, sol 60%)



# Inferencia difusa

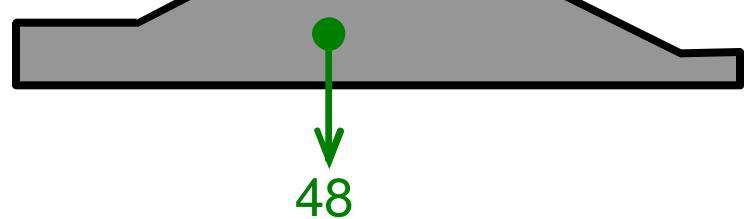
Pronóstico del clima ( temperatura:19º, sol 60%)



0.67 0.8 0.67

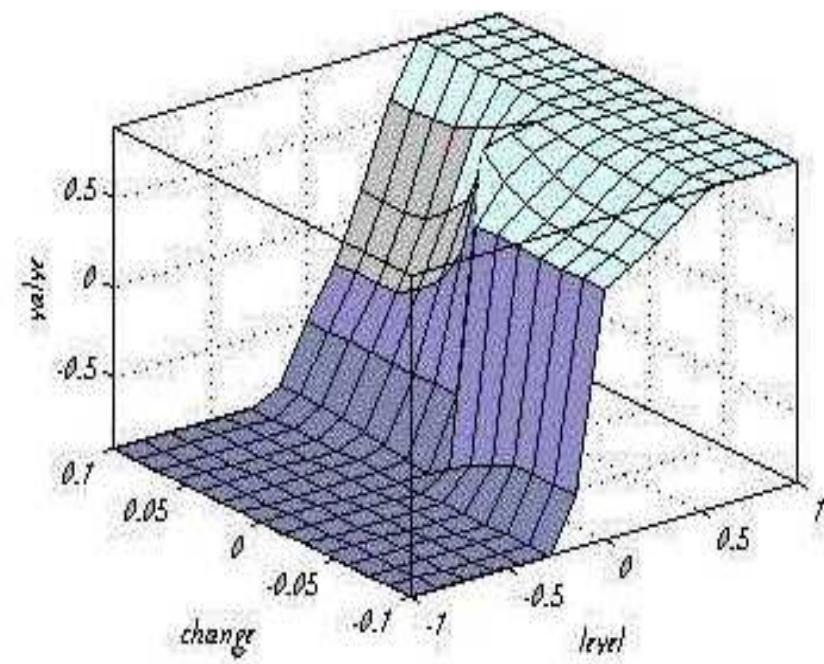
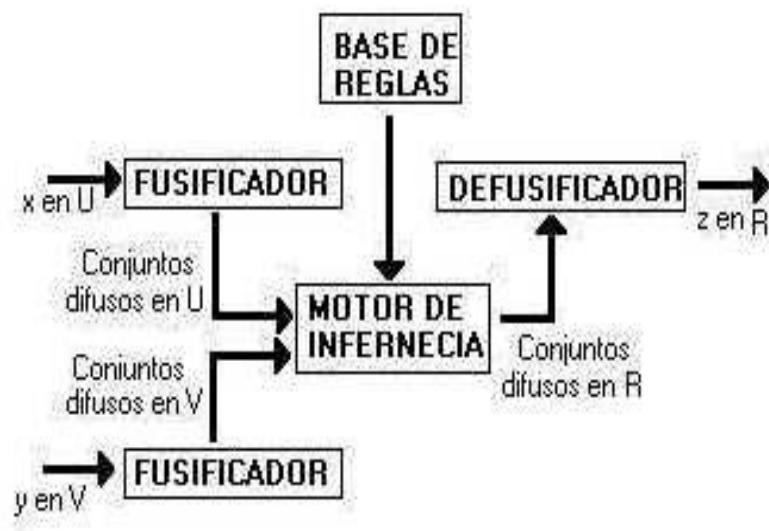
 OR 

0.33 0 0.33



Turistas(19,60) ≈ 48

# Sistemas difusos



# Teorema de aproximación universal

Suponga que el universo discurso de entrada U es cerrado y acotado en  $R^N$ . Entonces, para cualquier función continua  $g(x)$  en U y un valor  $\varepsilon > 0$ , existe un sistema difuso  $f(x)$  de la forma:

$$f(\mathbf{x}) = \frac{\sum_{l=1}^M y_l \left( \prod_{i=1}^N a_i^l \exp\left(-\frac{(x_i - x_i^l)^2}{(\sigma_i^l)^2}\right)\right)}{\sum_{l=1}^M \prod_{i=1}^N a_i^l \exp\left(-\frac{(x_i - x_i^l)^2}{(\sigma_i^l)^2}\right)}$$

Tal que :

$$\sup_{x \in U} |f(x) - g(x)| < \varepsilon$$

- L. X. Wang , *A course in fuzzy systems and control*, PTR 1996

# Bases difusas

$$f(x) = \frac{\sum_{l=1}^M y_l \left( \prod_{i=1}^N \mu_{A'_i}(x_i) \right)}{\sum_{l=1}^M \left( \prod_{i=1}^N \mu_{A'_i}(x_i) \right)}$$

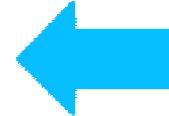


Expansión FBD

$$y(x) = f_s(x) = \sum_{l=1}^M y_l \cdot \phi_l(x)$$

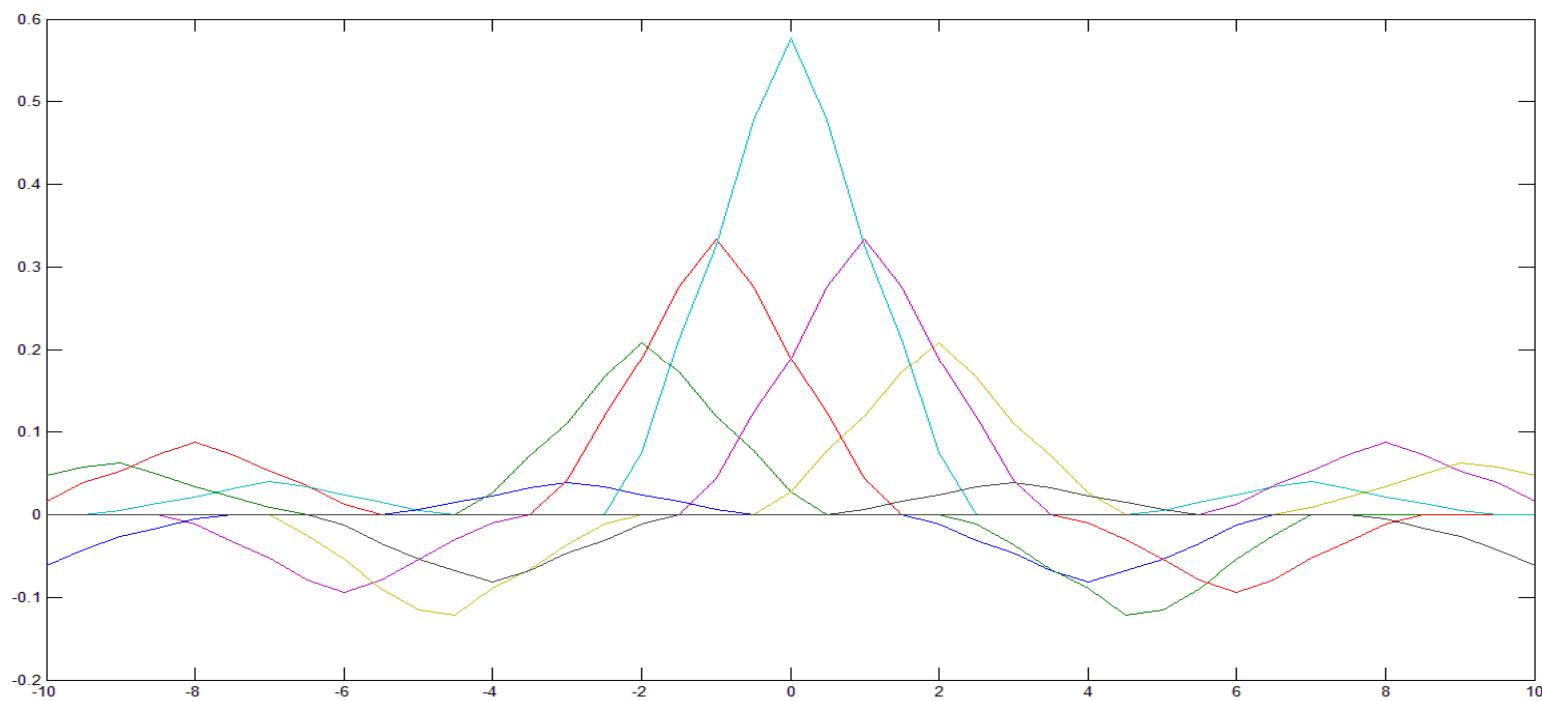
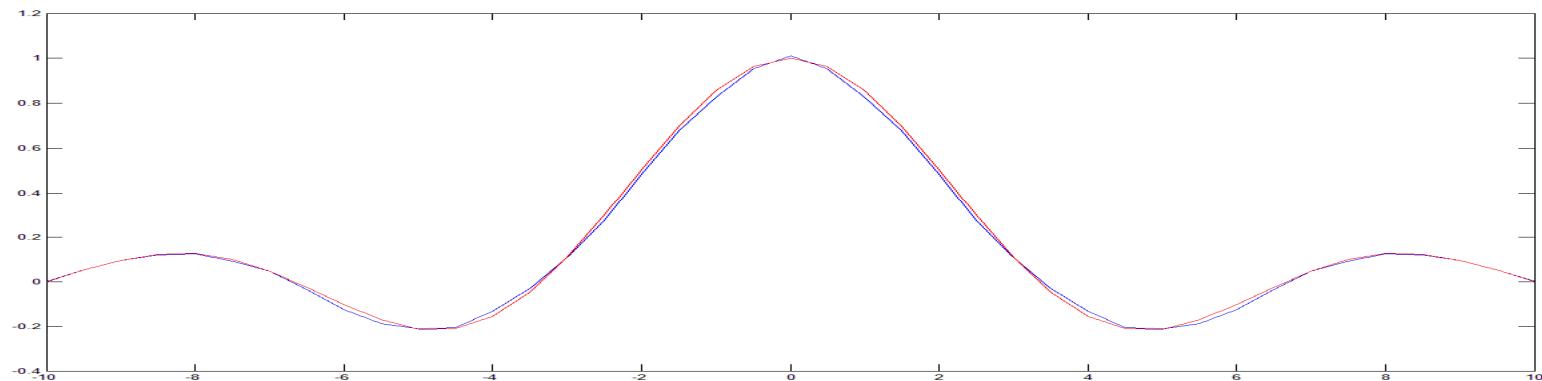


Funciones de base difusa

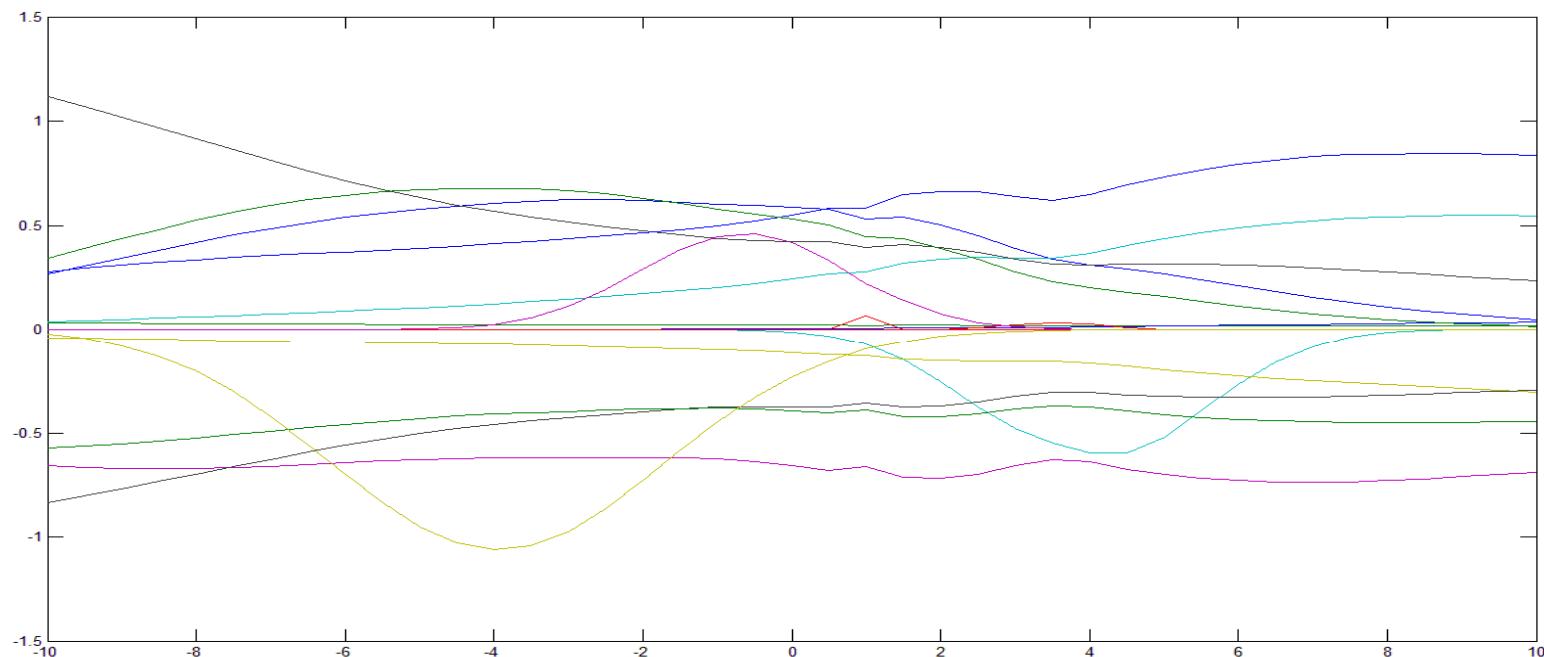
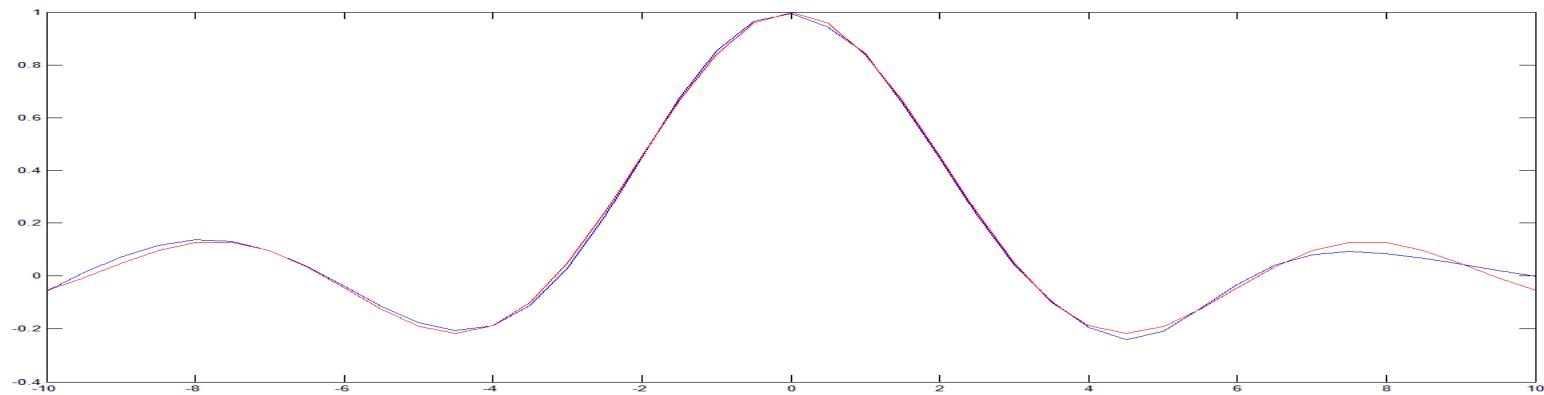


$$\phi_l(x) = \frac{\prod_{i=1}^N \mu_{A'_i}(x_i)}{\sum_{l=1}^M \left( \prod_{i=1}^N \mu_{A'_i}(x_i) \right)}$$

# Expansión de funciones de base difusa

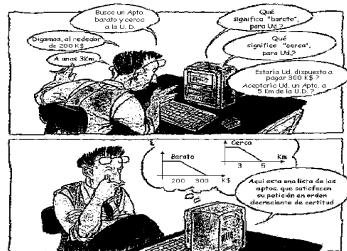
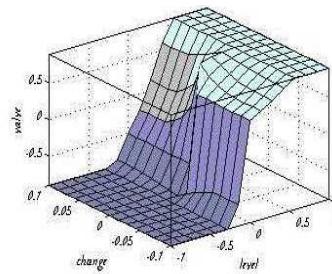


# Expansión de funciones de base difusa

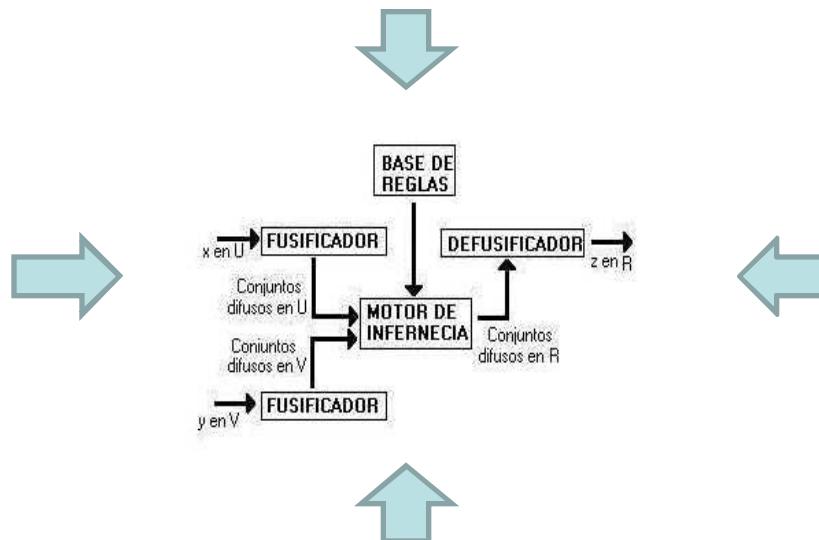


# Mensaje para la casa....

Mapas  
No lineales

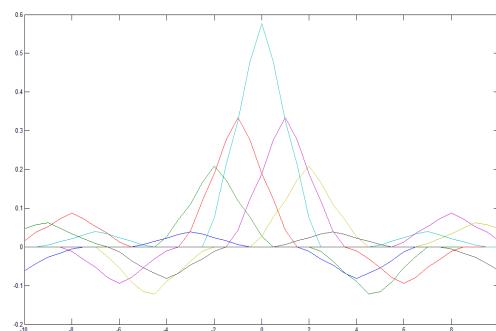


Reglas  
Y  
palabras



$$f(\mathbf{x}) = \frac{\sum_{l=1}^M y_l \left( \prod_{i=1}^N a_i^l \exp\left(-\frac{(x_i - x_i^l)^2}{(\sigma_i^l)^2}\right) \right)}{\sum_{l=1}^M \prod_{i=1}^N a_i^l \exp\left(-\frac{(x_i - x_i^l)^2}{(\sigma_i^l)^2}\right)}$$

Aproximador  
Universal

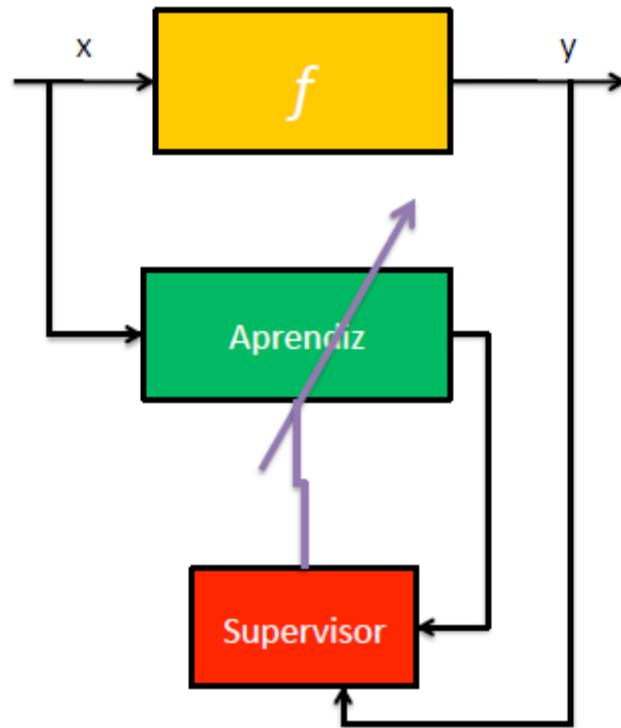


Expansión  
De bases

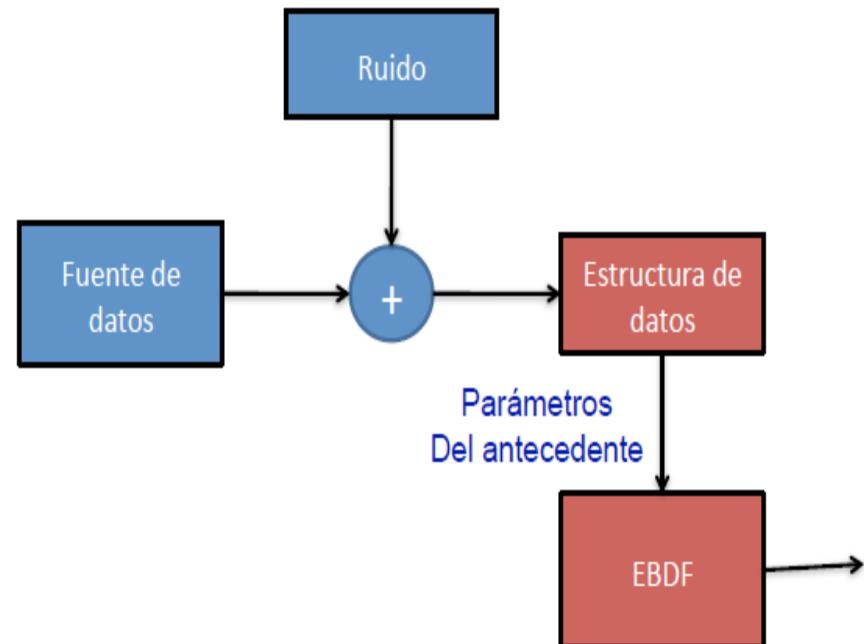
πάντα χωρεῖ καὶ οὐδὲν μένει (*todo cambia, nada permanece*)  
(*Heraclito*)

## **MODELADO DIFUSO EVOLUTIVO**

# Aprendizaje en sistemas difusos



Supervisado



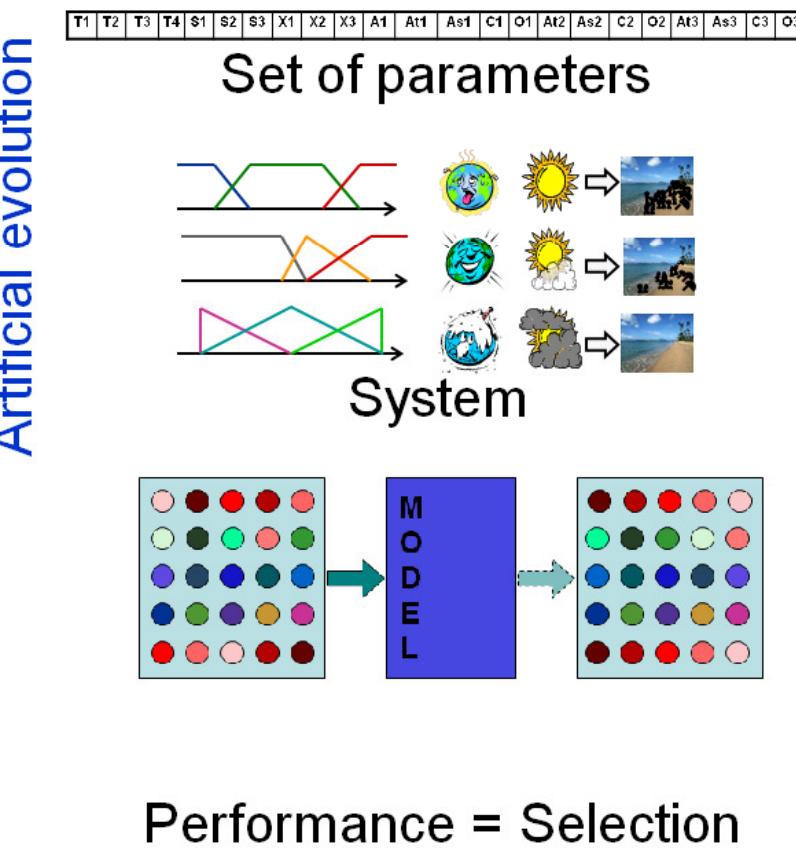
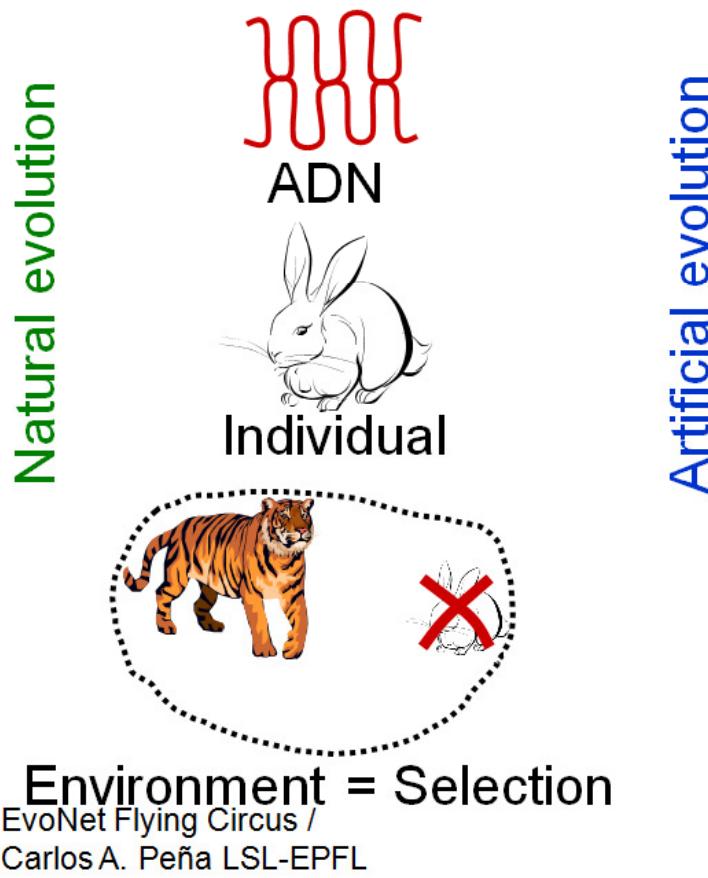
No supervisado

# Aprendizaje en sistemas difusos

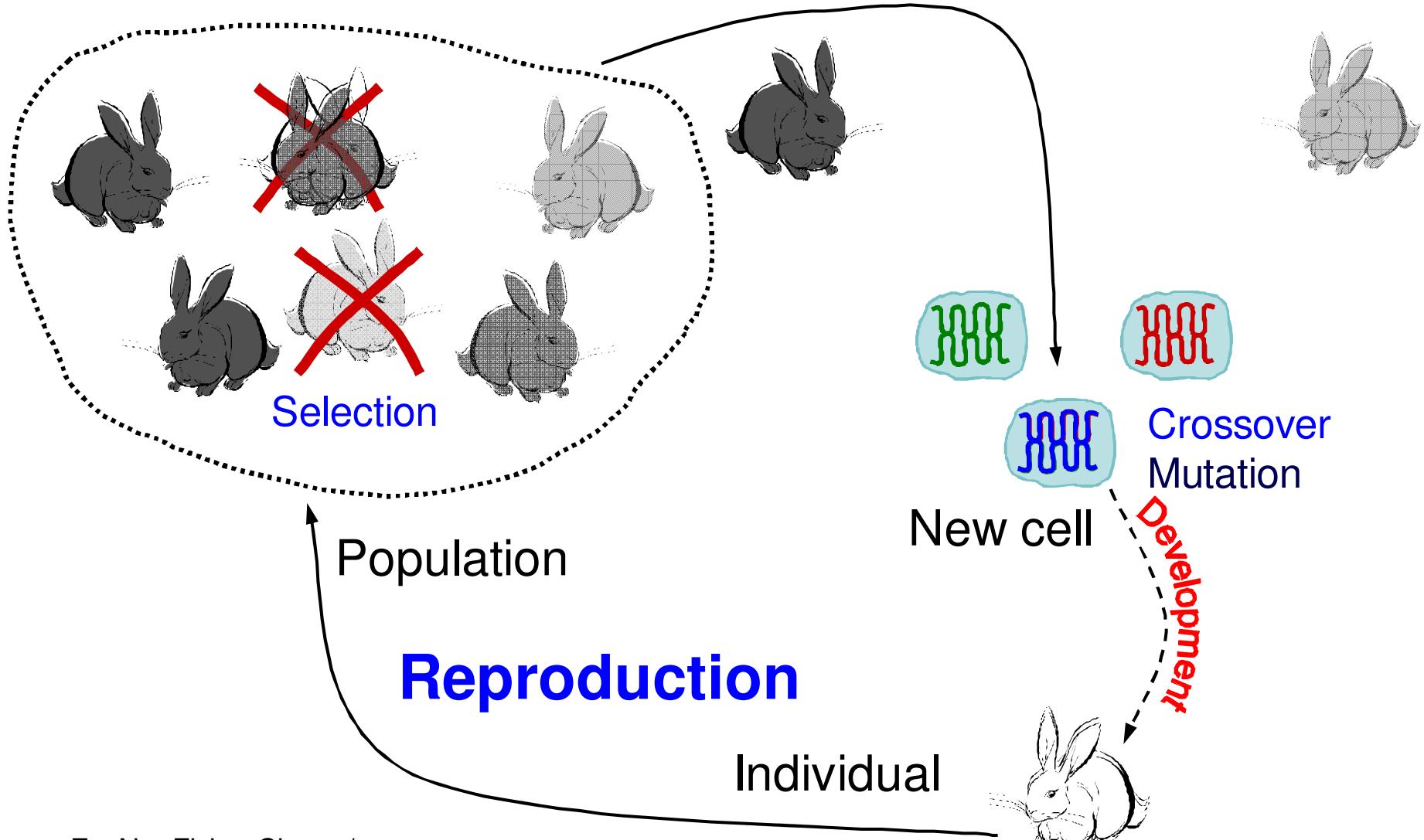
- Mínimos cuadrados
- Back-propagation
- Agrupamiento (C-means, K-means, etc)
- Mapas auto-organizados
- ....

# Evolución artificial

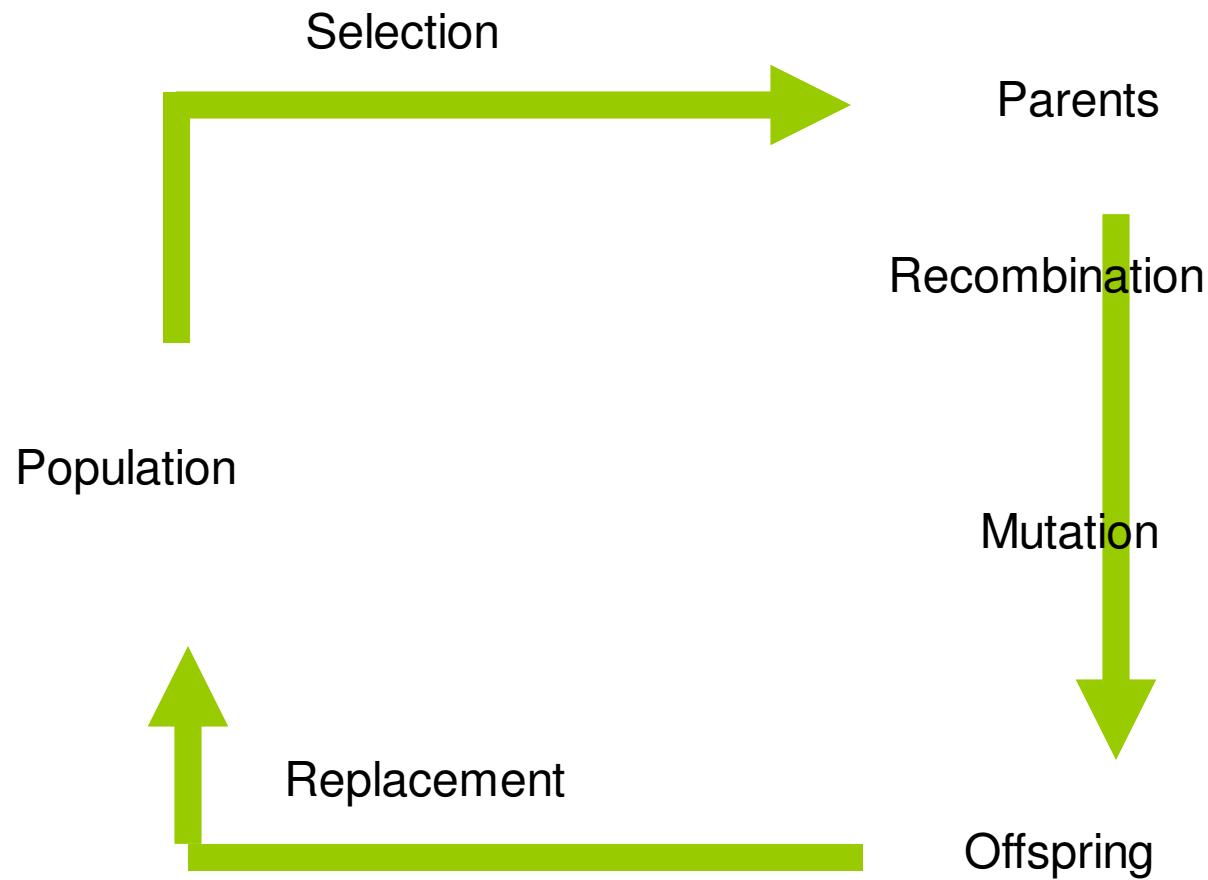
## Una metáfora de la evolución natural



# Un modelo de evolución...



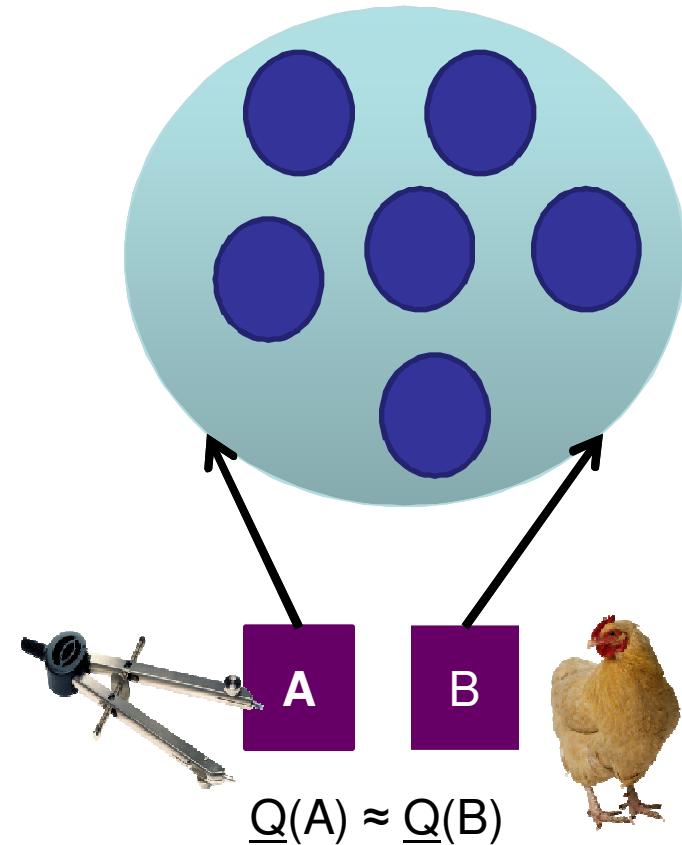
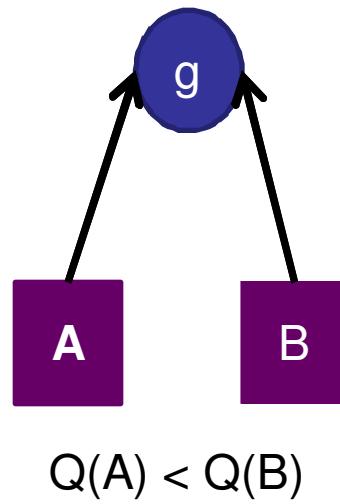
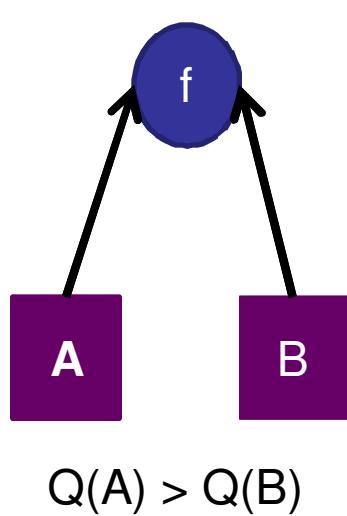
# El ciclo evolutivo



# Algoritmos evolutivos (generaciones)

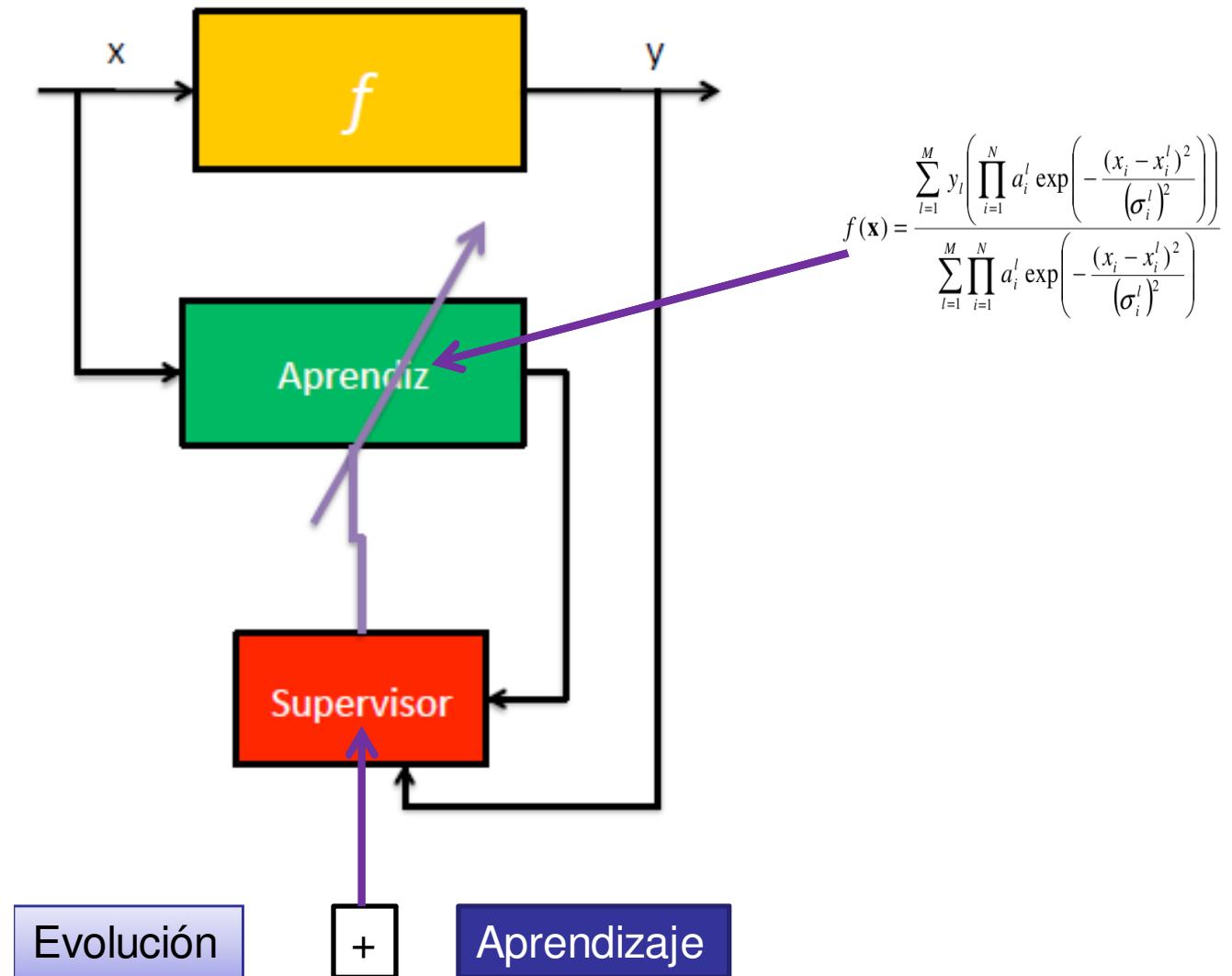
- Algoritmo genético simple
- Algoritmos co-evolutivos
- Evolución diferencial
- Algoritmo genético compacto
- Algoritmos meméticos
- Algoritmo simbióticos
- ....

# No free Lunch theorem



Wolpert, D.H.; Macready, W.G., "No free lunch theorems for optimization,"  
*Evolutionary Computation, IEEE Transactions on* , vol.1, no.1, pp.67,82, Apr 1997

# Sistema difuso evolutivo



“Experience: that most brutal of teachers. But you learn, my God do you learn.”

*C.S. Lewis*

## **CASOS DE APLICACIÓN**

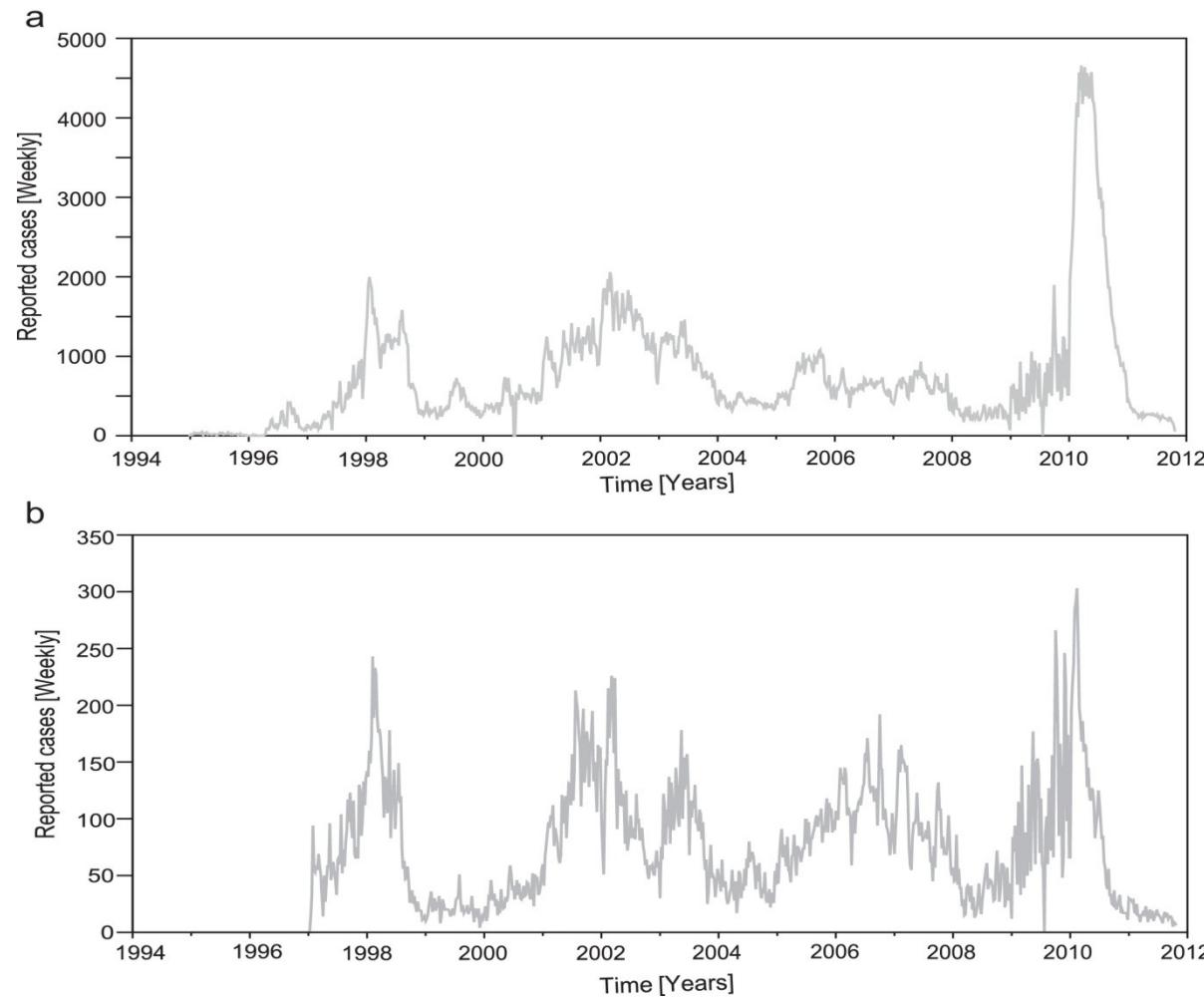
# Fenómenos complejos

- Fenomenos (señales o sistemas)
- Complejos (caos, multiples escalas, fractalidad, multi-fractalidad, propiedades emergentes, auto-organización,etc)

# Casos de aplicación

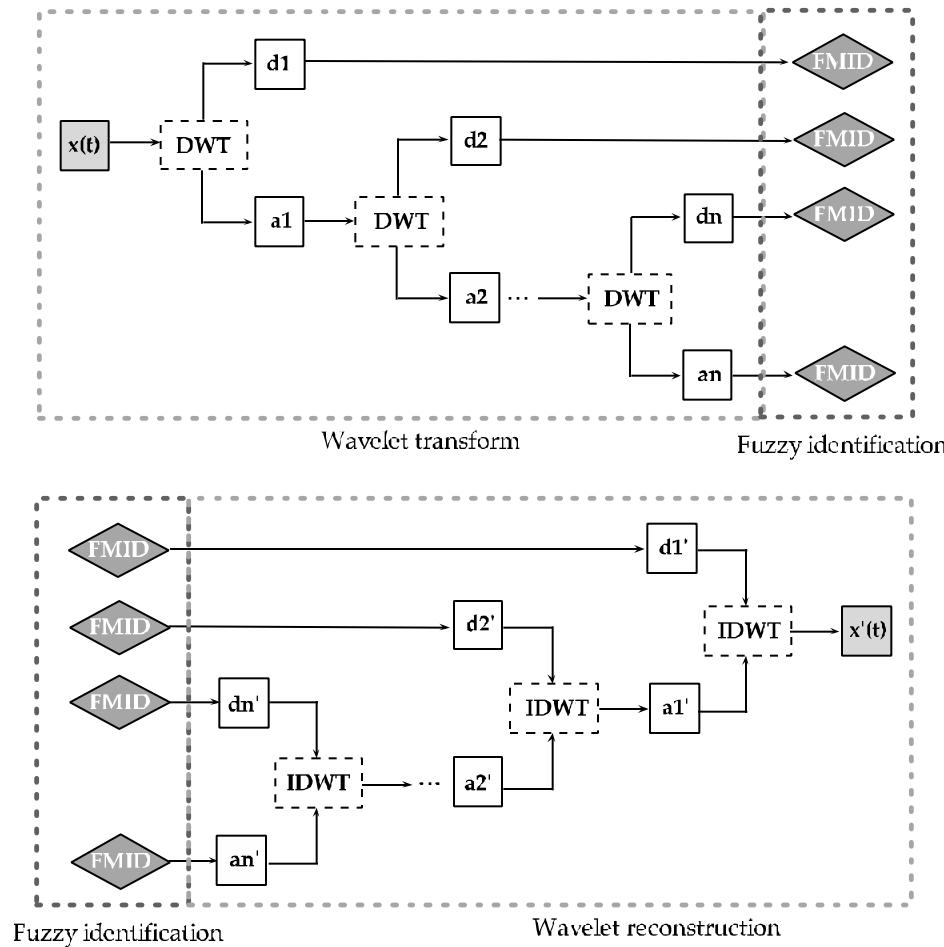
- Modelo difuso multi-escala de la epidemia de Dengue en Colombia.
- Clasificación de eventos PM10 en Bogotá DC.
- Auto-organización guiada de automatas celulares difusos.

# Modelo difuso multi-escala de la epidemia de Dengue en Colombia



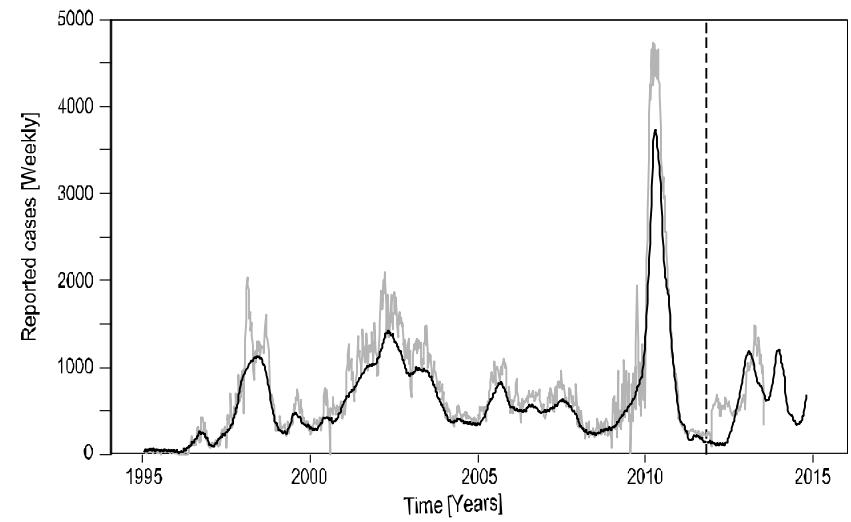
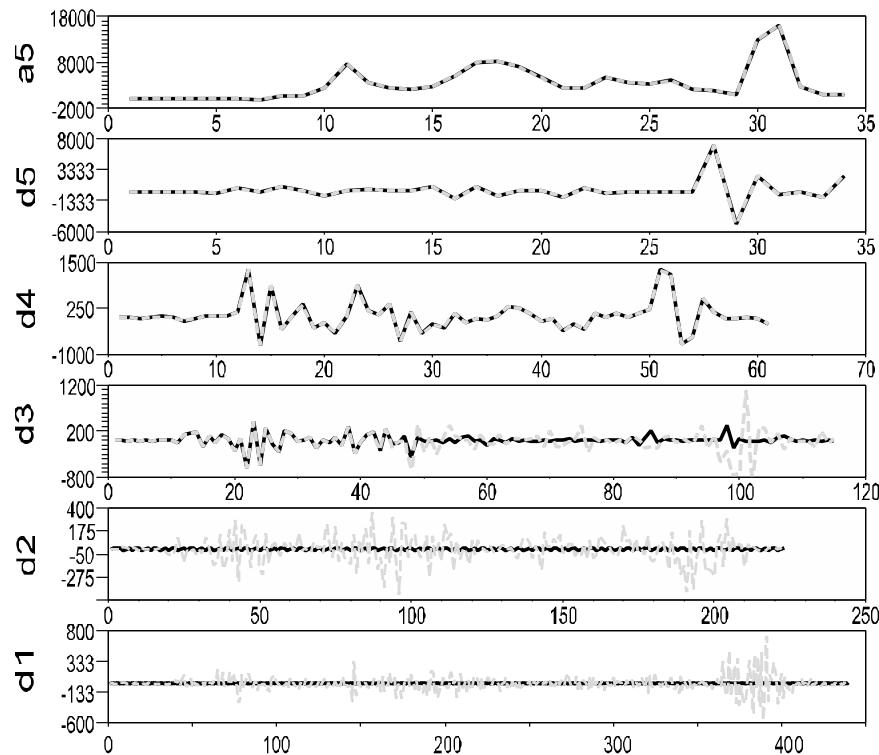
C. Torres, S. Barguil, M. Melgarejo, A. Olarte, *Fuzzy model identification of dengue epidemic in Colombia based on multiresolution analysis*, Artificial Intelligence in Medicine, 60,1,pp 41-51, Jan 2014

# Modelo difuso multi-escala de la epidemia de Dengue en Colombia



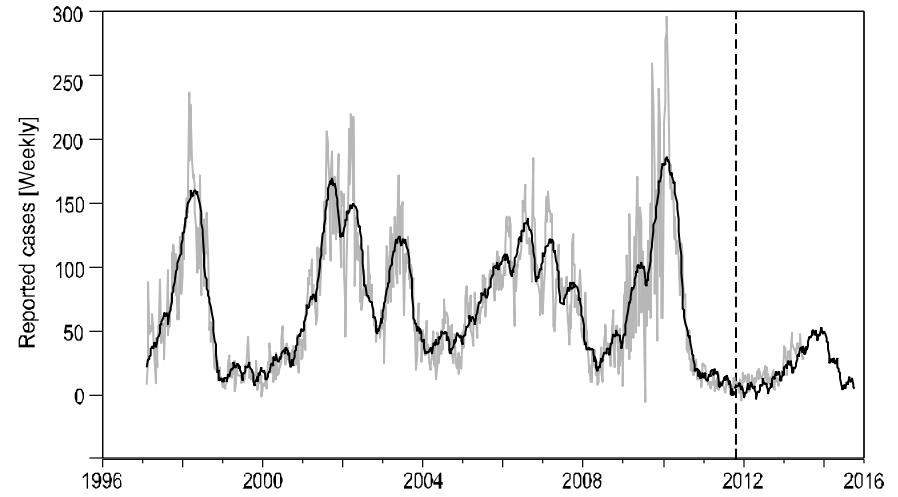
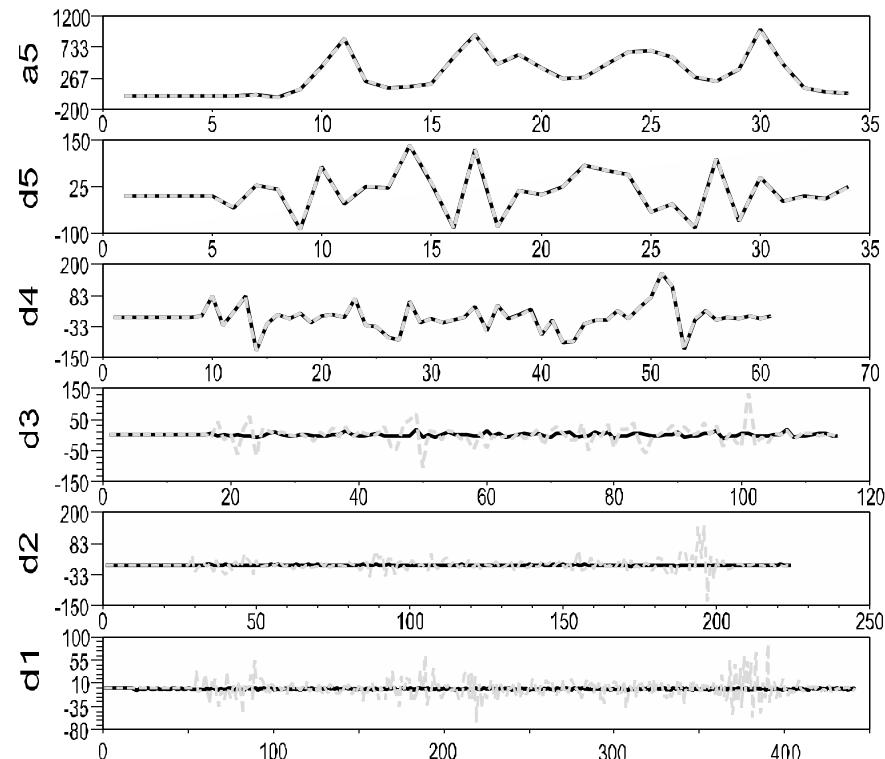
C. Torres, S. Barguil, M. Melgarejo, A. Olarte, *Fuzzy model identification of dengue epidemic in Colombia based on multiresolution analysis*, Artificial Intelligence in Medicine, 60,1,pp 41-51, Jan 2014

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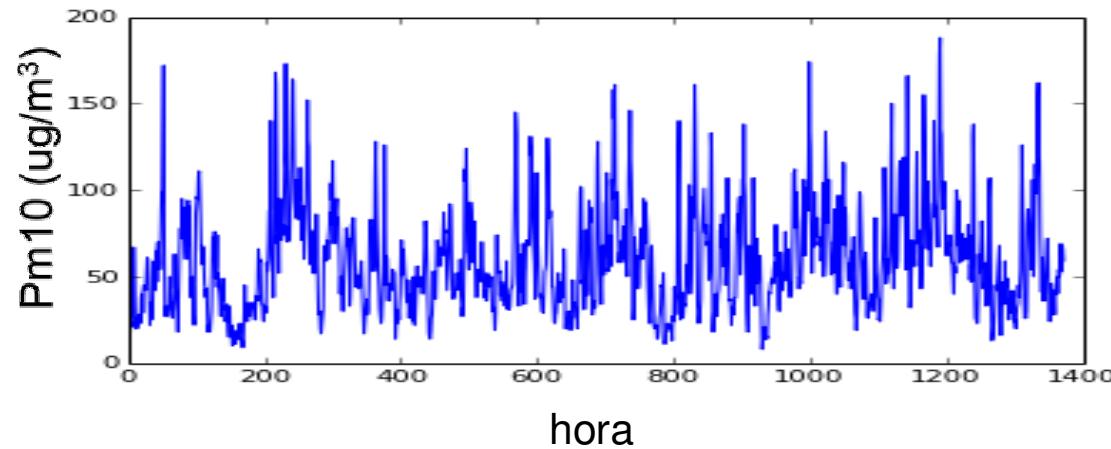
C. Torres, S. Barguil, M. Melgarejo, A. Olarte, *Fuzzy model identification of dengue epidemic in Colombia based on multiresolution analysis*, Artificial Intelligence in Medicine, 60,1,pp 41-51, Jan 2014

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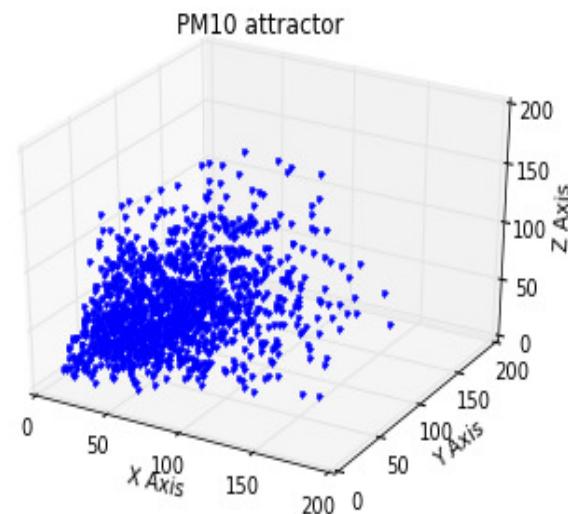


C. Torres, S. Barguil, M. Melgarejo, A. Olarte, *Fuzzy model identification of dengue epidemic in Colombia based on multiresolution analysis*, Artificial Intelligence in Medicine, 60,1,pp 41-51, Jan 2014

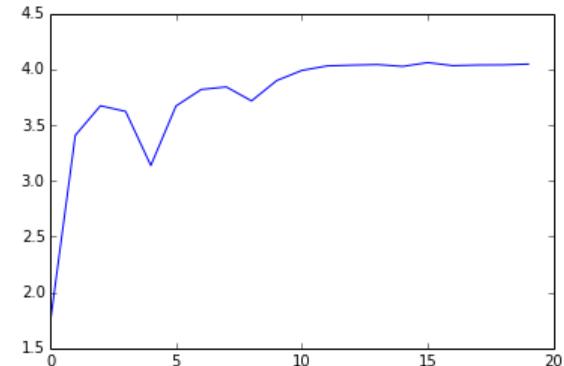
# Clasificación de eventos PM10 en Bogotá DC



Reconstrucción del  
Atractor  
(Teorema de Takens)

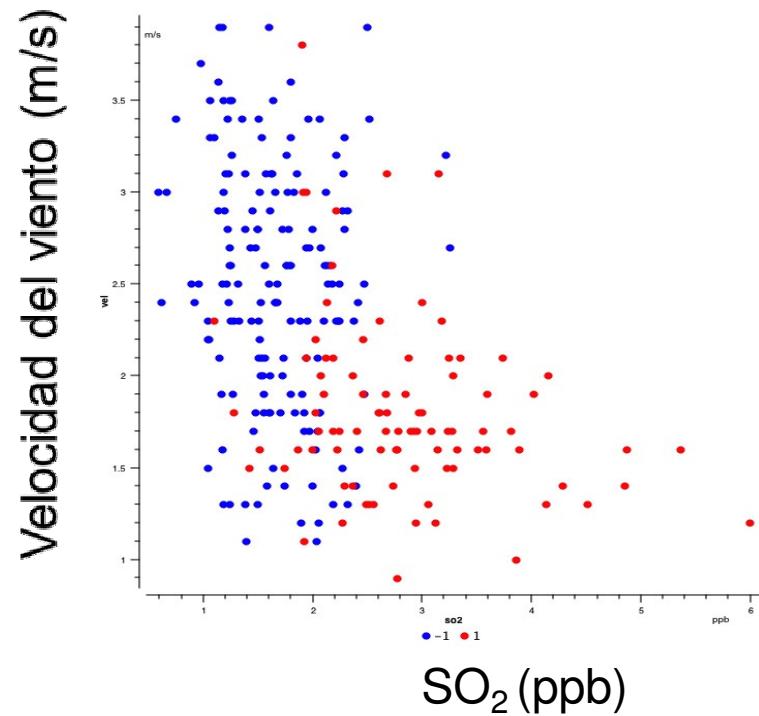
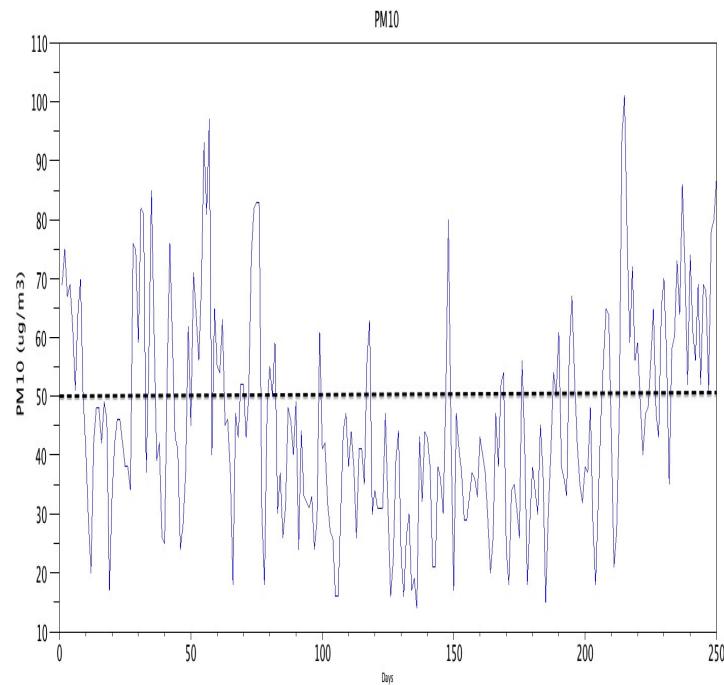


Máximo exponente de Lyapunov  
(Método de Rosenstein)



M. Melgarejo, N. Obregon, C. Parra, Applying Computational Intelligence to the Classification of Environmental Situations Associated to Pollution Events, *IEEE Transactions Latin America*, Dic 2014, (TBP)

# Clasificación de eventos PM10 en Bogotá DC



M. Melgarejo, N. Obregon, C. Parra, Applying Computational Intelligence to the Classification of Environmental Situations Associated to Pollution Events, *IEEE Transactions Latin America*, Dic 2014, (TBP)

# Clasificación de eventos PM10 en Bogotá DC

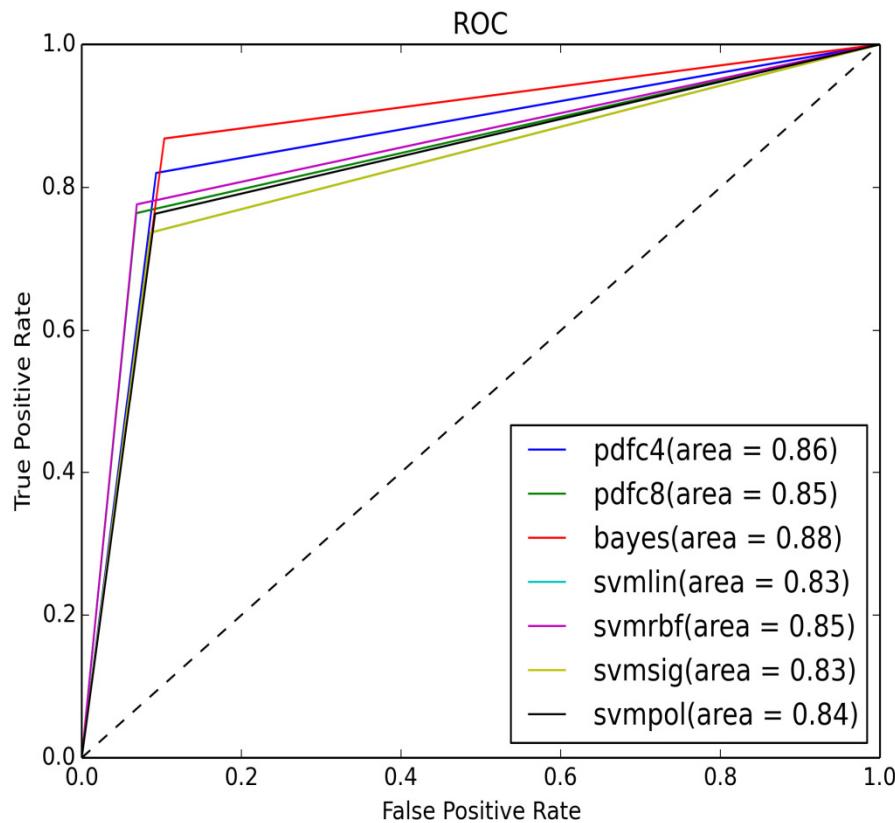
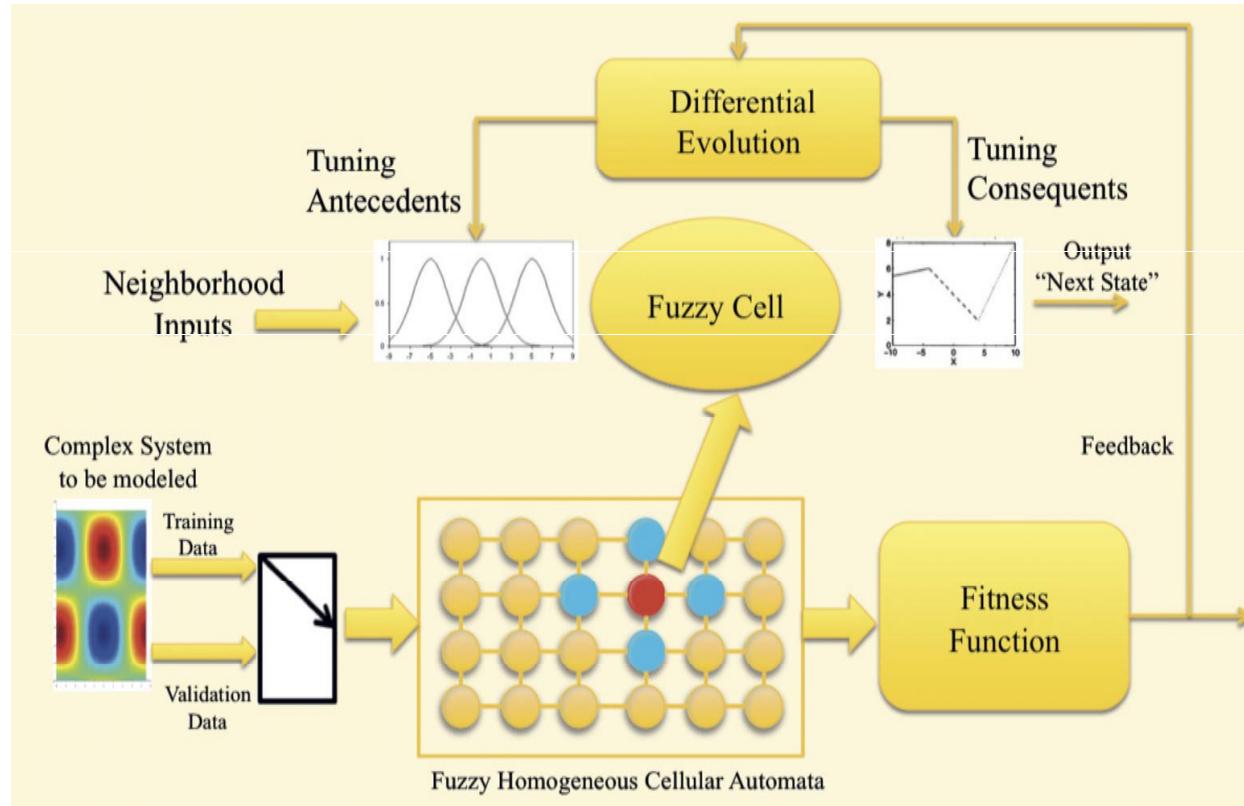


TABLA I  
RESULTADOS DE LA VALIDACIÓN DE LOS CLASIFICADORES

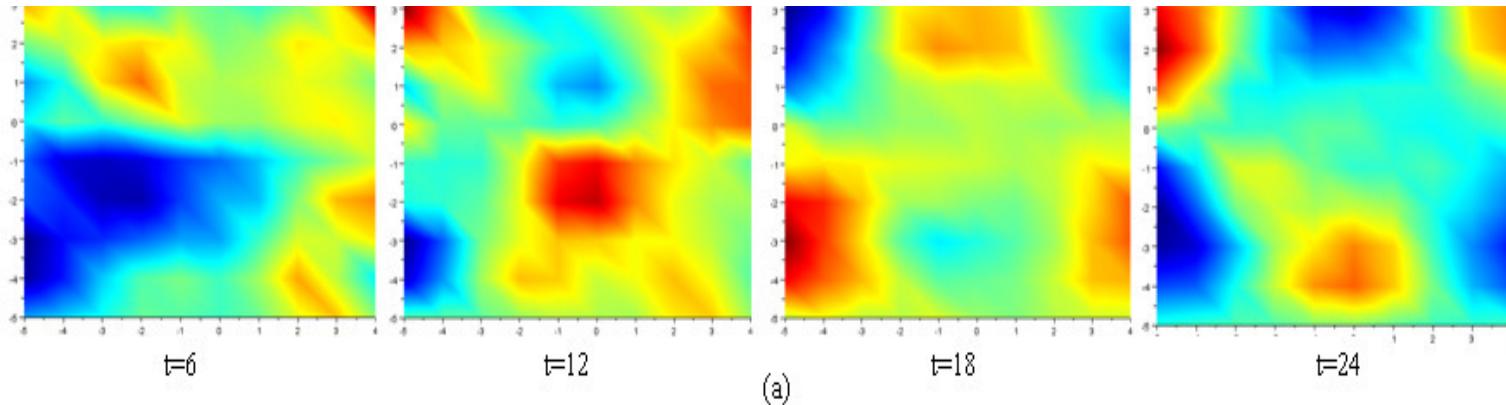
Clasificador	Exactitud (%)	Sensibilidad (%)	Especificidad (%)
Bayes	85.63	91.1	75.31
Svmlin	86.44	93.02	74.04
Svmpol	86.41	93.16	73.69
Svmrbf	86.40	92.98	74.00
Svmsig	86.47	93.00	74.15
Pdfc4	84.16	85.10	82.81
Pdfc8	84.86	85.60	84.50

# Automatas celulares difusos (Auto-organización guiada)

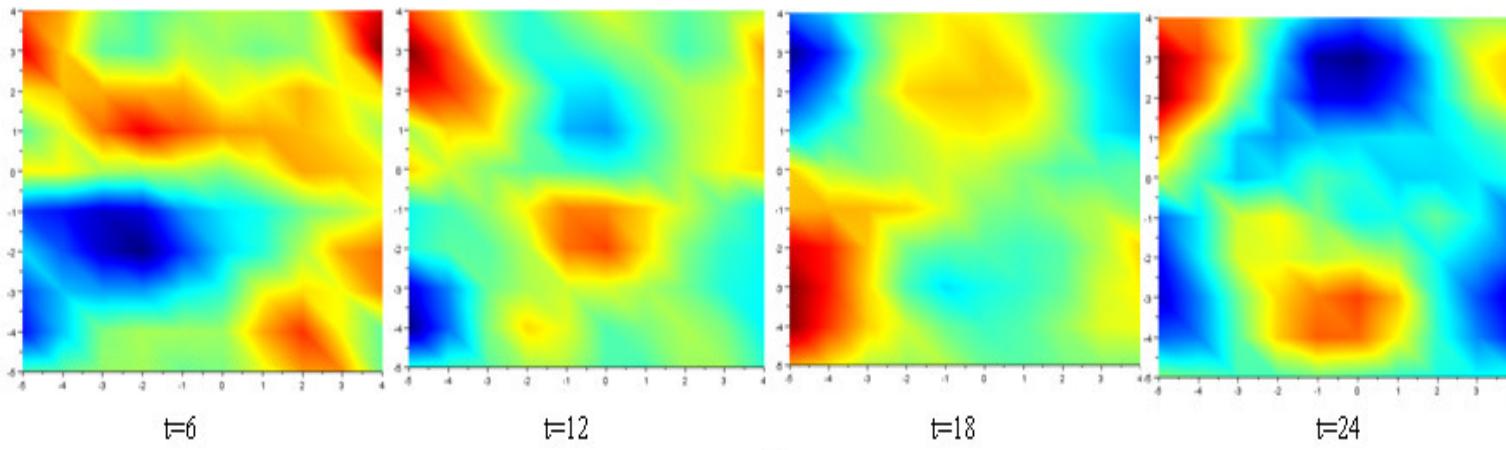


Chivata, J.; Luengas, Y.; Melgarejo, M.; Obregon, N., "Complex system modeling using TSK fuzzy cellular automata and differential evolution," 2013 IEEE International Conference on Fuzzy Systems (FUZZ), July 2013

# Automatas celulares difusos (Auto-organización guiada)



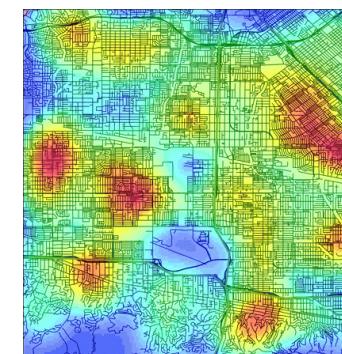
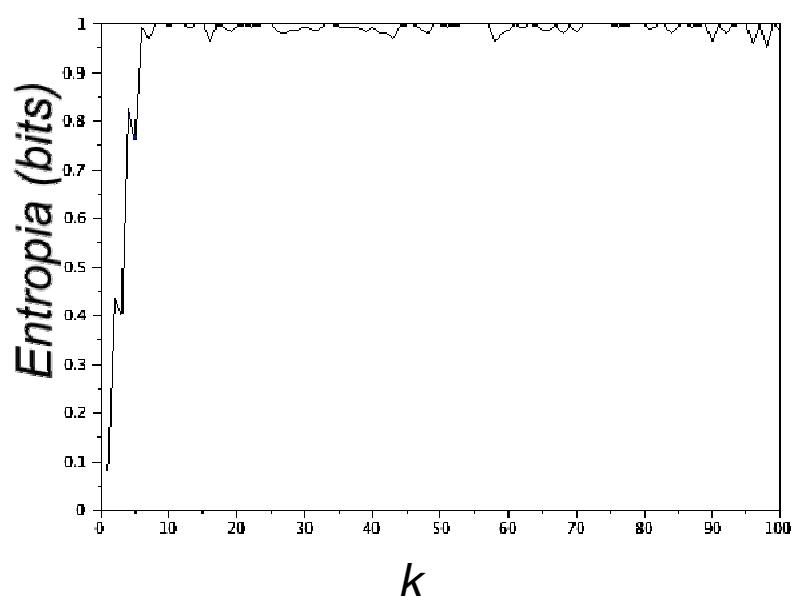
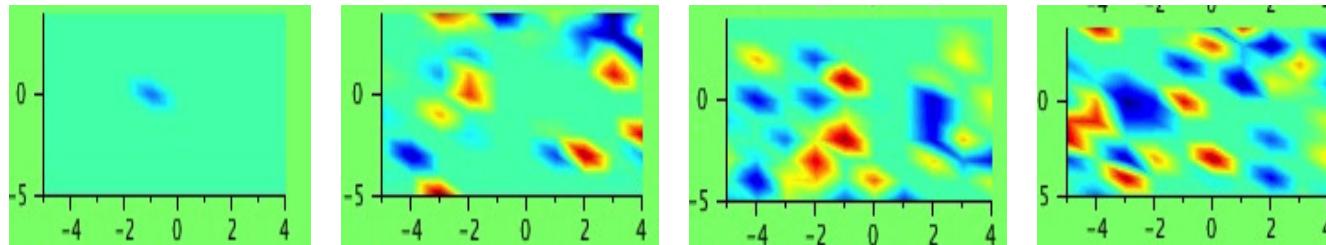
(a)



(b)

Chivata, J.; Luengas, Y.; Melgarejo, M.; Obregon, N., "Complex system modeling using TSK fuzzy cellular automata and differential evolution," *2013 IEEE International Conference on Fuzzy Systems (FUZZ)*, July 2013

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Martin B. Short, P. Jeffrey Brantingham, Andrea L. Bertozzi, and George E. Tita, **Dissipation and displacement of hotspots in reaction-diffusion models of crime**, PNAS 2010 ;, February 22, 2010,

M. Melgarejo, N. Obregon, "On the design of fuzzy cellular automata following a maximum entropy principle," *2014 IEEE Conference on Norbert Wiener in the 21st Century (21CW)*, pp.1,5, 24-26 June 2014

# Para finalizar...

- Modelado difuso evolutivo : una herramienta.
- Limitaciones: Teorema de aproximación universal y No Free Lunch Theorem.
- Posibilidades: conexión Micro-Macro.

*Evaluando en lo Macro, diseño en lo Micro.*